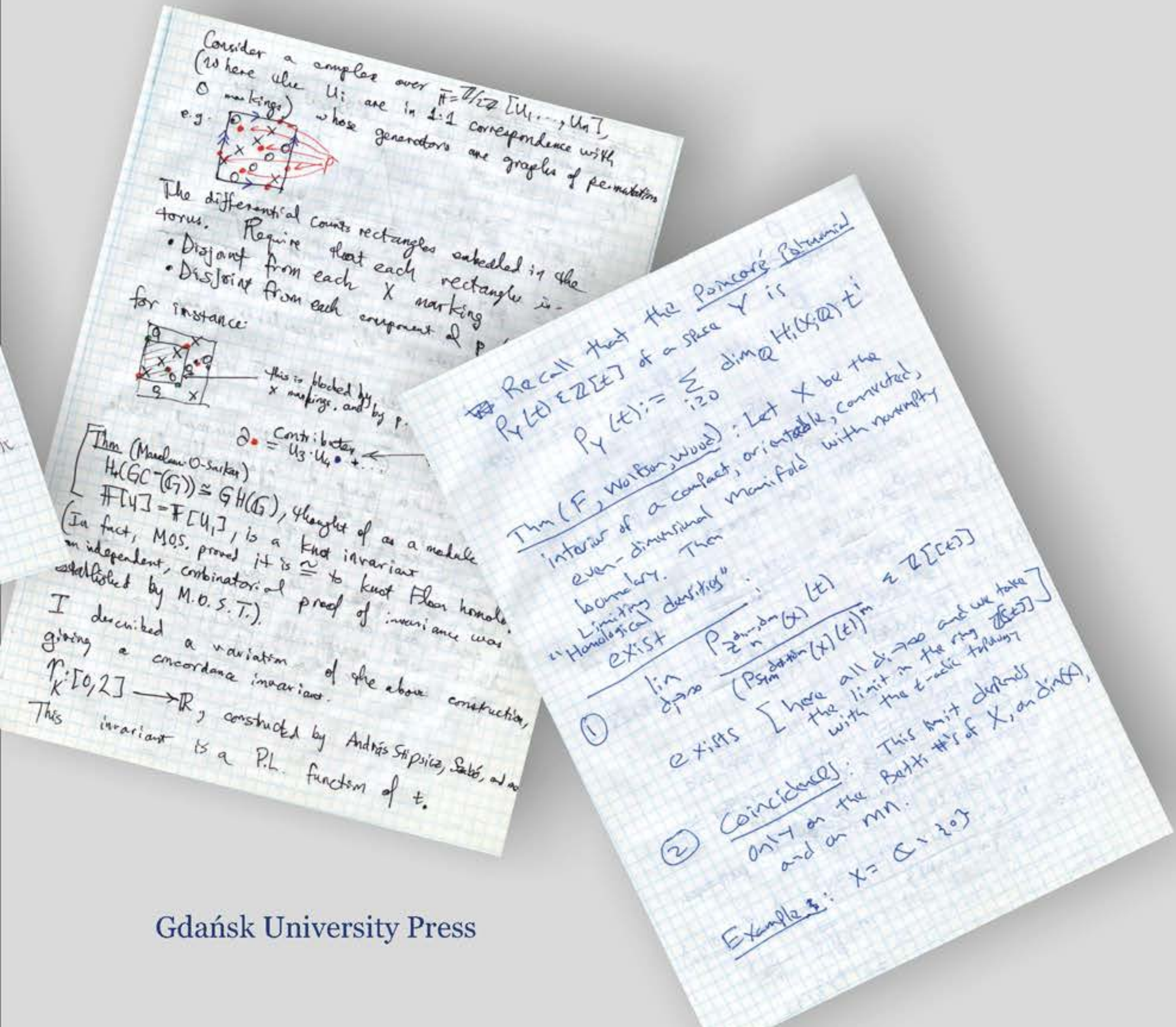
Andrzej Jankowski Memorial Lectures

Gdańsk 1999–2018

Edited and with an Introduction
by Andrzej Szczepański



Gdańsk University Press

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Introduction

The Andrzej Jankowski Memorial Lectures, held annually since 1999, occupy an important place in the scientific life of Gdańsk and the University of Gdańsk. This is a lecture on mathematical subjects, given by world-famous mathematicians representing the best mathematical centers in the world.

The idea of the Andrzej Jankowski Memorial Lectures was born in Edinburgh at the house of Andrew Ranicki in 1998. The first lecture was delivered by Graeme Segal from Cambridge University, who has known Jankowski. Before the lecture in May 1999, professor Zbigniew Ciesielski suggested to me to write some book which should enclose personal abstracts of the main talks.

The memorial lecture was always followed by a workshop on subjects related to the mathematical interest of the main speaker. Usually on this miniconferences the second talk was given by the main speaker. Always it was a lectuer different from the main one.

From the beginning the Andrzej Jankowski Memorial Lectures have a Scientific Committee whose members were Kazimierz Gęba (1999–2000), Andrzej Granas (1999–2003), Andrew Ranicki (1999–2018) and Stefan Jackowski (1999–2013). Today, the Scientific Committee consists of three members: Tomasz Mrówka (2014–), Józef Przytycki (2014–) and Andrzej Szczepański (2002–). The main lectures were given outside Gdańsk two times only. The first one was given in Cracow during the Conference on Algebraic Topology 26th June — 1st July 2005, and the lecture was delivered by Tomasz Mrówka. The second one was given in Warsaw 6–11th July 2009, also during the Conference on Algebraic Topology. That time the speaker was Mladen Bestvina.

Starting from 2004, the organizers prepared the listeners of the main lecture through a three day long preparatory conference. The first one was held in Cracow in May 2004, before lecture VI. The next ones were held in Św. Lipka in 2007, 2008, 2009 and 2011 before the lectures IX, X, XI and XIII. Preparatory conferences to the lectures XVI and XVII took place in Toruń, and to the lectures XIX and XX they took place in Cracow again. Finally, we mention preparatory conferences in Olsztyn, 2013 and in Sandomierz, 2015.

The lectures are given by remarkable mathematicians, including Fields' medalist Maxim Kontsevich. Most of the lectures were organised by the Institute of Mathematics of Gdańsk University.

We present a copy of handwritten abstracts of all talks. They are also presented as printed texts, which were made from the handwritten ones by Bartosz Putrycz and Rafał Lutowski. The Andrzej Jankowski Memorial Lectures were financial supported by Institute of Mathematics of Gdańsk University, Institute of Mathematics of Polish Academy of Science, the Polish National Science Center and city of Gdańsk. The current website of the Andrzej Jankowski Memorial Lectures has the address: mat.ug.edu.pl/ajml.

Editorial note

Some of the texts were corrected by the authors, so the printed versions of them are partly different from the handwritten abstracts. During proofreading necessary changes in punctuation and standardization were made. The texts were standardized in the following aspects: enumerations and dates, references were enclosed to the texts with footnotes, where it was possible abbreviated forms were limited and replaced with full names, underlined words were substituted by italic, also irregular use of italic was stabilized.

Andrzej Szczepański

String theory and the space of surfaces

Graeme Segal (Cambridge University) 21st May, 1999

One of the central problems of fundamental physics is to produce a quantum theory of gravity. It seems likely that a solution will involve some generalization of the notion of space-time as a smooth manifold. String theory, although it is at present only the beginnings of a theory, is the most interesting candidate-theory. One can think of it as an analogous programme to "non-commutative geometry", which starts from the observation that to give a smooth manifold M is equivalent to giving the ring $C^\infty(M)$ of smooth functions on M, and then generalizes the commutative ring $C^\infty(M)$ to a non-commutative one.

In string theory $C^\infty(M)$ is generalized to a much more surprising and subtle object, namely a 2-dimensional conformal field theory. This statement, however, needs to be made more precise in two ways. On one hand, such a field theory is meant to correspond not to a general Riemannian manifold, but to one which satisfies Einstein's gravitational equations. On the other hand, the field theory encodes not just the classical manifold, but an infinitesimal neighborhood of it in a space of objects analogous to manifolds which do not satisfy the gravitational equations.

A 2-dimensional field theory can be defined as a rule which associates a vector space $\mathcal{H}_S$ to each compact oriented 1-manifold S, and a linear map $U_\Sigma : \mathcal{H}_{S_0} \to \mathcal{H}_{S_1}$ to each oriented cobordism Σ from S_0 to S_1. We require $\mathcal{H}_{S_0 \sqcup S_1} \cong \mathcal{H}_{S_0} \otimes \mathcal{H}_{S_1}$, and also that $U_{\Sigma'} \circ U_\Sigma = U_{\Sigma' \cup \Sigma}$ when we have a composition of cobordisms. If U_Σ depends only on the smooth structure of Σ we speak of a "topological" field theory; if U_Σ depends in addition on a conformal structure on Σ we have a conformal field theory. In the topological case the vector space $\mathcal{H}_{S^1}$ is a commutative Frobenius algebra with multiplication

$U_\Sigma : \mathcal{H}_{S^1} \otimes \mathcal{H}_{S^1} \to \mathcal{H}_{S^1}$ given by a "pair of pants" Σ. It turns out that this algebraic structure completely encodes the combinatorial topology of the sewing-together of surfaces. A conformal field theory is accordingly a generalization of a commutative ring in which we have not just one multiplication but a family of multiplications parametrized by the conformal structures on a pair of pants Σ. In the case of the theory corresponding to a Riemannian manifold M, the space $\mathcal{H}_{S^1}$ is the space of differential forms of a certain kind on the loop space of M.

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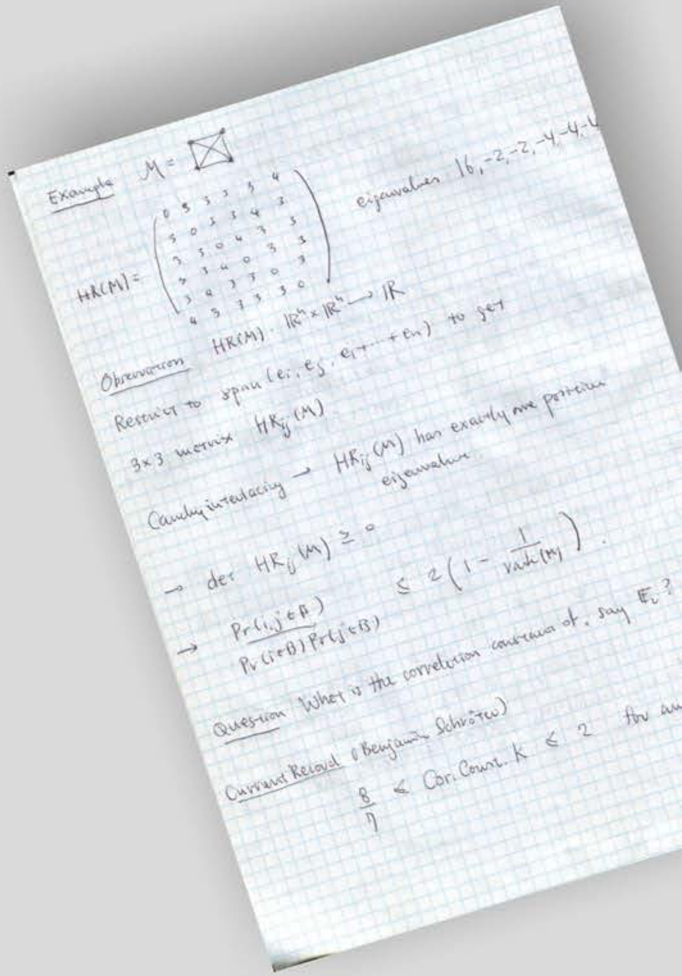
In string theory $C^\infty(M)$ is generalized to a much more surprising and subtle object, namely a 2-dimensional conformal field theory. This statement, however, needs to be made more precise in two ways. On one hand, such a field theory is meant to correspond not to a general Riemannian manifold, but to one which satisfies Einstein's gravitational equations. On the other hand, the field theory encodes not just the classical manifold, but also an infinitesimal neighbourhood of it in a space of objects which play the role of manifolds which do not satisfy the gravitational equations.

A 2-dimensional field theory can be defined as a rule which associates a vector space $\mathcal{H}_s$ to each compact oriented 1-manifold S, and a linear map $U_\Sigma \colon \mathcal{H}_{S_0} \to \mathcal{H}_{S_1}$ to each oriented cobordism Σ from S_0 to S_1. We require $\mathcal{H}_{S_0 \amalg S_1} \cong \mathcal{H}_{S_0} \otimes \mathcal{H}_{S_1}$, and also that $U_{\Sigma'} \circ U_\Sigma = U_{\Sigma' \cup \Sigma}$, when we have a composition of cobordisms. If U_Σ depends only on the smooth structure of Σ, we speak of a "topological" field theory; if U_Σ depends in addition on a conformal structure on Σ we have a conformal field theory. In the topological case the vector space $\mathcal{H}_{S^1}$ turns out to be a commutative Frobenius algebra with multiplication $U_\Sigma \colon \mathcal{H}_{S^1} \otimes \mathcal{H}_{S^1} \to \mathcal{H}_{S^1}$ induced by a "pair of pants" cobordism Σ. It turns out that this algebraic structure completely encodes the combinatorial topology of the sewing-together of surfaces. A conformal field theory is accordingly a generalisation of a commutative ring in which we have not just one multiplication but a family of multiplications parametrized by the conformal structures on a pair of pants Σ. In the case of the theory corresponding to a Riemannian manifold M, the space $\mathcal{H}_{S^1}$ is the space of differential forms of a certain kind – they have semi-infinite degree – on the loop space of M.



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